The Investment-Sales Sensitivity: The Link between Tangible and Intangible Capital

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Abstract

The extant literature has documented that the sensitivity of corporate investment in tangible capital to cash flow has declined to zero, while the sensitivity of investment to sales has been strong among US manufacturing firms. However, the cause of these remain unexplored. This paper presents a model in which the information about future productivity of a firm is contained in its sales, but the durability of intangible capital is random, which makes the investment in intangible capital random. Cash flow, which is sales minus various costs including the investment in intangible capital, therefore loses its strong connection with the investment in tangible capital and with future cash flow. This is more relevant for firms that heavily rely on intangible capital. Empirical evidence supports the implications of the model.

Key words: Investment-sales sensitivity, investment-cash flow sensitivity, cash flow predictability, new economy, operating leverage.

JEL codes: D22, D25, G31

1. Introduction

There has been an extensive empirical literature on corporate investment decisions. A large part of it centers on the discussion of the investment-cash flow sensitivity (ICFS), which is the finding that corporate investment depends on cash flow for manufacturing firms in the US, in face of the poor empirical performance of Tobin's Q represented by the market-to-book assets ratio. A debate lasted for more than twenty years early this century between two schools of thoughts in explaining the investment-cash flow sensitivity. One school, represented by Fazzari, Hubbard and Petersen (1988) who discovered the investment-cash flow sensitivity, asserts that it is indicative of the existence of financial constraint. The other school, initiated by Poterba (1988) and Kaplan and Zingales (1997), contends that investment depends on cash flow because the current cash flow contains information about future cash flows and serves as a proxy for marginal productivity, so the existence of the investment-cash flow sensitivity is consistent with the classical Q-theory proposed by Tobin (1969). The debate subsided gradually, however, as the investment-cash flow sensitivity declined over time and disappeared around the year 2000. In a recent paper, Zhang (2023) document that there is an investment-sales sensitivity during the period 1972-2021 for the manufacturing firms in the US. Unlike the investment-cash flow sensitivity, the investment-sales sensitivity remains economically and statistically strong throughout the sample period. Grullon and Ikenberry (2025) find the usual average Q does much better in explaining investment when sales are used as a control variable because firms with low sales tend to have excess capacity which makes investment unnecessary.

While the role of sales in explaining corporate investment in tangible capital has been established, the mechanism through which sales drive investment is still not clear. In particular, why sales can do the job, while cash flow can no longer, remains unknown. Based on the preliminary data patterns documented in this paper, I develop a model to explain the reasons behind the strength of the investment-sales sensitivity and the decline of the investment-cash flow sensitivity. The model is rooted in the neoclassic economic theory of corporate investment and is broadly consistent with the Q-theory, under the assumption that marginal product of capital is unobservable, similar to the models of Alti (2003) and Moyen (2004) in explaining the investment-cash flow sensitivity. To highlight the difference between sales and cash flow, the model focuses on the role of the investment in intangible capital through the rising operating leverage and its volatility, especially for the new-economy firms with intensified competition among them. The model implies that investment depends on sales due to its informational advantage over cash flow in this context. In addition, the model describes the economic rationale for sales to guide investment, especially for new-economy firms with high expected value and variation of operating leverage.

To understand what makes cash flow and sales behave differently in the later years of the sample, I contrast firms of different features: NASDAQ-listed vs. NYSE/AMEX-listed firms, and high-tech vs. low-tech firms. The results show that the difference between the investment-cash flow sensitivity and the investment-sales sensitivity manifests more among NASDAQ-listed and high-tech firms, which represent new-economy firms. It is more for new-economy firms than for old-economy firms that future cash flow is difficult to predict, but sales played more important role than cash flow itself to predict future cash flow. It is more for new-economy firms than for old-economy firms that investment shifted its dependence on cash flow to its dependence on sales. More specifically, I examine the source of the variation between sales and cash flow, which causes the difference between the investment-sales sensitivity and the investment-cash flow sensitivity. The examination is based on the accounting identity that Sales (SA) equal the sum of cost of goods sold (COGS), selling, general and administrative expenses (XSGA), miscellaneous items involving taxes, interest payments etc. (Misc) and cash flow (CF). The loss of sensitivity of investment to cash flow is largely derived from the volatile nature of XSGA, which can be regarded as investment in intangible capital.

The analyses from the above angles shows a clear picture. In the early years of the sample, old-economy firms dominate, so both cash flow and sales provide useful guidance to the decisions on investment in tangible capital. These firms are mostly listed on NYSE or AMEX, and are mostly in low-tech industries. Over the time, however, more and more new-economy firms enter the sample. More such firms are listed in NASDAQ and many of them are in high-tech industries. For most new-economy firms, making profits right away after they are listed on exchanges is a luxury. The priority for them is to quickly expand and to increase their market share by increasing their sales. In competing with their peers, they are often forced to spend more heavily on advertisement, research and development, and perks for retaining talents, in order to build up their intangible/organizational capital. All of these go into XSGA, which reduces cash flow for given sales. As a result, cash flow for these new-economy firms becomes more volatile and less self-predictable In the meantime, investment in tangible capital of such firms are made in line with the future demand of their products, predicted by current sales. In this sense, the dependence of investment in tangible capital on sales is from the fundamentals channel, as well as from the information channel. In a nutshell, there are alignments among sales, the tangible capital stock, and intangible capital stock, but, due to random shocks to the durability of existing intangible capital, the connection between investments in tangible and intangible capital is loose. Cash flow, which is the residual after deducting the investment in intangible capital as well as cost-of-goods-sold from sales, gradually loses its connection with the investment in tangible capital when new-economy firms occupy a larger share of the economy.

The rest of the paper is organized as follows. Section 2 introduces data and reports the features of the key variables used in the paper. Section 3 presents a model featured by the difference between old-economy firms and new-economy firms in terms of the productivity of intangible capital and the volatility of the durability of intangible capital. Section 4 presents empirical results, consistent with the mode predictions. Section 5 concludes. The derivation of the model and the definition of high-tech industry are relegated to the Appendixes.

2. Data

2.1. Sample

The sample used in this paper is the public US firms in the COMPUSTAT annual file from 1975 to 2024, excluding those in the financial industry (SIC codes from 6000 to 6799) and the public administration industry (SIC codes from 9100 to 9999). The typical sample in the literature of investment-cash flow sensitivity is the US manufacturing firms (SIC codes from 2000 to 3999). The empirical results of this paper indeed hold more strongly for manufacturing firms. But they also hold true for Agriculture, Forestry & Fishing (SIC codes from 0100 to 0999), Mining (SIC codes from 1000 to 1499), Construction (SIC codes from 1700 to 1799), Transportation & Public Utilities (SIC codes from 4000 to 4999), Wholesale Trade (SIC codes from 5000 to 5199), Retail Trade (SIC codes from 5200 to 5999) and Services (SIC codes from 7000 to 8999).

One of the challenges in conducting the work is to differentiate old-economy firms and new-economy firms. While the meaning of these terms is evident, there is no single clear identifier to accomplish the task, so we will rely on multiple, imperfect criteria. These criteria involve the exchanges on which the firms are listed, the industries they belong to, and, in future work, various firm characteristics which broadly differ across old- and new-economy firms.

The sample is divided between firms listed on NYSE/AMEX and those listed on NASDAQ. NYSE and AMEX, founded in the 18th and 19th centuries respectively, are known for older, more established companies with stringent listing conditions, while NASDAQ, established in 1971, is home to many newer companies focused on technology and innovation, with less stringent listing conditions for profitability.

The sample is also divided between high-tech firms and low-tech firms based on the 3-digit SIC industries they belong to. Table A1 in Appendix lists all the 3-digit industries that are classified as high-tech industries. The definition basically follows those used in the literature. The table also provides examples of firms within these industries. The firms outside these high-tech industries are classified as low-tech firms.[1]

[1] The definition is not perfect, as firms in the same 3-digit industries may produce very different products and use different processes to produce. Tesla, for example, belongs to the motor vehicle industry, and is classified as a low-tech firm because the motor vehicle industry has existed for a long time. Less strikingly but more pervasively are many firms like Nike, which belongs to apparel industry, a traditional low-tech industry, but is making all kinds of high-tech gadgets related to sports and health.

2.2. Key Variables

Investment in tangible capital, Inv, is the capital expenditure (COMPUSTAT item, CAPX) of a firm-year (i, t), scaled by the total assets (AT) at the end of last year. Inv is the variable used in the literature of the investment-cash flow sensitivity for manufacturing firms, extended in this study to cover broader industries.

SA is the total sales of a firm-year (i, t) scaled by the beginning-of-the-year AT. SA minus cost of goods sold (COGS) is known as gross profit (GP). GP minus selling, general, and administrative expenses (XSGA) is known as operating income before depreciation (OI). According to standard classifications, COGS consists of production costs that can be traced to specific products, and therefore represents variable costs, while XSGA consists of costs that are overhead costs, untied to specific products, and therefore represents fixed costs. OI minus a few miscellaneous items (Misc) is the cash flow (CF) measure adopted in the literature of the investment-cash flow sensitivity.[2] The accounting relationships among these variables are

$$\begin{aligned} \text{SA} &= \text{COGS} + \text{GP} & (1) \\ &= \text{COGS} + \text{XSGA} + \text{OI} & (2) \\ &= \text{COGS} + \text{XSGA} + \text{Misc} + \text{CF}. & (3) \end{aligned}$$

Figure 1 plots the annual time-series of cross-sectional averages of SA, GP, OI and CF in lines and those of COGS, XSGA and Misc as distances between lines. All of these variables are scaled by the total assets last year. The cross-sectional averages are taken with the cross-sectional distributions winsorized at the 1% level on both sides.

Figure 1 here

The figure shows that average sales per dollar of total assets declined over time, though non-monotonically. It started slightly below 2 in the late 1970s and dropped to around 1 in early

[2] The miscellaneous items, Misc, is the sum of several items including interest expenses (XINT), nonoperating income (NOPI), special items (SPI), taxes (TXT), and minority interest (MII).

2020s. The patterns look similar across all four sub-samples. Of the components of sales, the average of cost-of-goods-sold experienced the most evident decline. The average XSGA tends to be higher for NASDAQ firms than for NYSE/AMEX firms and higher for high-tech firms than for low-tech firms. Misc, the difference between OI and CF, is a small fraction of total assets on average. Around 2000, the average OI and average CF for NASDAQ firms and for high-tech firms became negative and remained negative for the rest of the sample period. All these patterns have some bearings to the similarity and differences between the investment-cash flow sensitivity and the investment-sales sensitivity, to be explained later.

Among the above-mentioned variables, XSGA deserves a special attention, as it plays two different roles in the analysis. In the sales decomposition from the accounting perspective, XSGA is regarded as the fixed cost, cost that is unrelated to the level of production, deducted from sales like COGS to calculate operating income and cash flow. It is expensed periodically without carrying on to the next period. In the more recent economics and finance literature, however, XSGA is regarded as the investment in intangible/organizational capital, although the latter is not an official accounting variable and how XSGA gets accumulated to form intangible/organizational capital lacks a solid principle.[3]

2.3. Variance Decomposition: A Tale of Two Costs

In order to understand what causes cash flow to behave so differently from sales, we look at the accounting relationships (1)-(3). We compare two costs items: COGS and XSGA, as two components of SA. For a given variable x in firm-year it, let the sample variance calculated by its past ten years (from year $t-9$ to year t, under the condition of at least 5 non-missing observations) be denoted as $\mathrm{Var}_{it}(x)$. For a pair of variables x, y, the sample covariance is calculated similarly. We calculate Var_{it}(SA), Var_{it}(COGS), Var_{it}(XSGA), Var_{it}(GP), Var_{it}(OI), Var_{it}(COGS/SA), Var_{it}(XSGA/SA), Cov_{it}(COGS, GP) and Cov_{it}(XSGA, OI) and discuss them below.

[3]In the economics and finance literature, scholars have used the perpetual inventory method with an economy-wide, time-invariant depreciation rate to construct the firm-level organization capital from assumed initial values.

Figure 2 is the scatter plot of Var_{it}(COGS) against Var_{it}(SA) and Figure 3 is the scatter plot of Var_{it}(XSGA) against Var_{it}(SA) for NYSE/AMEX, NASDAQ, low-tech and high-tech sectors. The difference between Var_{it}(COGS) and Var_{it}(XSGA) in their relation to Var_{it}(SA) can also be seen from these scatter plots. Var_{it}(COGS) is much more aligned with Var_{it}(SA) than Var_{it}(XSGA) does. Overall, Var_{it}(XSGA) is higher for NASDAQ and high-tech sectors than NYSE/AMEX and low-tech sectors, respectively, on average and especially so for low Var_{it}(SA) firm/years.

Figure 2 here

Figure 3 here

To see the impact of variation in XSGA on cash flow, we focus on two equations: SA=COGS+GP and GP=XSGA+OI. Figure 4 plots the time series of the cross-sectional averages of Var_{it}(SA), Var_{it}(COGS), and Var_{it}(GP) for the four sectors. Like the averages of SA_{it}, COGS_{it}, and GP_{it} themselves, the average sample variances for the NYSE/AMEX firms decline over time. Others display a rising trend in the first half of the sample period. The sample variances of NASDAQ and high-tech firms tend to be higher than their NYSE/AMEX and low-tech counterparties, but they also decline faster, especially in the last 20 years of the sample period. Overall, for all exchanges and both low- and high-tech sectors, the sample variances of SA, COGS and DP at the end of the sample period are much lower than those at the beginning of the sample period. The sample covariance between COGS and DP, calculated using past ten years of data for each firm-year, has a slightly positive cross-sectional average. As a result, on average, the sum of the sample variances of COGS and DP is smaller than the sample variance of SA.

Figure 4 here

Figure 5 plots the time series of the cross-sectional averages of Var_{it}(GP), Var_{it}(XSGA), and Var_{it}(OI) for the four sectors. Apart from similar patterns observed in the previous figure, a strikingly different pattern is that the sum of $\text{Var}_{it}(\text{XSGA})$ and $\text{Var}_{it}(\text{OI})$ is greater than $\text{Var}_{it}(\text{GP})$, especially for NASDAQ and high-tech firms. In fact, in the last few years of the sample period, the average $\text{Var}_{it}(\text{XSGA})$ of the high-tech firms has exceeded $\text{Var}_{it}(\text{GP})$. Correspondingly, the cross-sectional average of the sample covariance between XSGA and OI became negative in the later part of the sample period. [4]

Figure 5 here

2.4. A Synthesis of Data Patterns

The plots of the previous subsections reveal a few patterns which shed light to the background to understand the empirical evidence, to be presented later, that corporate investment depends on sales and, to a decreasing extent over time, on cash flow. These patterns can be summarized as follows.

Relative to old-economy firms, new-economy firms rely less on tangible capital and more on intangible capital. The former is directly observed from investment in, and the total stock of, tangible capital relative to assets. The latter is seen from investment in intangible capital relative to both assets and investment in tangible capital.

On average, sales contain the most reliable information about the productivity of a firm. Cost of goods sold is more or less a stable component of sales. As a result, gross profit also contains much information about productivity. However, the so-called fixed cost, XSGA, is in fact quite volatile, especially for new-economy firms. This makes operating income and cash flow, which are profit after the fixed costs, very volatile and less informative for future profitability. Because of this, operating income and cash flow provide less information than sales in guiding corporate investment decisions.

[4] The pattern is more obvious in the manufacturing industry. In the last few years of the sample period, the averages of $\text{Var}_{it}(\text{XSGA})$ and $\text{Var}_{it}(\text{OI})$ in NASDAQ and high-tech firms both exceed that of $\text{Var}_{it}(\text{GP})$.

3. A Model of the Investment-Sales Sensitivity

To understand the working of the investment-sales sensitivity and the declining investment-cash flow sensitivity, we develop a simple model to show what determines the productive capital structure, how investments in tangible capital and intangible capital are related to the productive capital levels, and why the investment-sales sensitivity prevails, while the investment-cash flow sensitivity withers.

The model is cast in a discrete-time framework with variables mapped to the basic accounting relations as much as needed. The model emphasizes the key difference that characterizes old-economy and new economy firms. For notational brevity, however, the model is written for a firm with its interactions with other firms deemphasized and the firm-specific parameters unlabeled. One of the goals in the model is to highlight the difference between cash flow and sales in their roles of guiding investment decisions.

A firm in the model is financed by equity only at its birth and no further financing from new investors is considered. Taxes are assumed to be zero for simplicity. The firm is represented by its production function and evolution of its state variables. The sales, S_t, of the firm from the end of year $t-1$ and the end of year t is determined by

$$S_{t+1} = A_{t+1} K_t^{\alpha} N_t^{\beta} L_{t+1}^{\theta}, \qquad \alpha, \beta, \theta > 0, \quad \alpha + \beta + \theta < 1, \tag{4}$$

where A_{t+1} is a comprehensive measure of the firm's productivity, K_t is the firm's tangible capital stock at the end of year t, N_t is its intangible capital stock, and L_{t+1} represents other productive factors (flow variables) such as labor and raw materials, assumed to be rented or purchased at competitively determined cost w at $t+1$ after A_{t+1} is observed. Let $a_t = \log A_t$ with $a_{t+1} = \eta + \delta a_t + \varepsilon_{t+1}$, where $\varepsilon_{t+1} \sim N(0, \sigma_{\varepsilon}^2)$, $\delta > 0$, and $\alpha + \beta + \theta + \delta = 1$. Therefore, sales are linearly homogeneous in (A_t, K_t, N_t, L_{t+1}). In terms of (K_t, N_t, L_{t+1}), however, the model features decreasing returns to scale.

The two types of capital evolve as

$$K_{t+1} = \rho K_t + I_{t+1}, \qquad \rho \in (0,1) \tag{5}$$

$$N_{t+1} = \nu_t N_t + J_{t+1}, \qquad \nu_t \in (0,1), \tag{6}$$

where the durability of tangible capital, ρ, is a constant and the durability of intangible capital, ν_t, is a random variable, identically and independently distributed across t. I_{t+1} and J_{t+1} are the investment in tangible capital and the investment in intangible capital respectively, decisions made after observing ν_t and A_{t+1}. The formulation above assumes away the adjustment cost, as it does not play an important role here. The initial capital stocks are therefore unimportant, they are adjusted to the optimal level immediately when the firm is established.[5]

The randomness of the durability of intangible capital is new in this paper, which may come from various sources. For example, the value of the firm's intellectual property may be partially destroyed by a new development of its competitors, an unexpected PR incident may occur, which hurts the reputation, or the CEO may suddenly become sick, hurting the firm's organizational capital. These things make ν_t lower than its expected value. There can be pleasant surprises as well, making ν_t higher than expected. As will be shown below, variations in ν_t will induce inverse variations in J_t for new-economy firms. Empirical evidence presented in the last section that the volatility of XSGA has increased over time for NASDAQ and high-tech firms can be a result of the fact that typical new-economy firms have higher variations. Condition (iii) is a crucial condition for addressing the issue of the reliance of tangible capital investment on cash flow and on sales. The randomness of intangible capital makes new-economy firms riskier.[6]

All the parameters (and therefore state variables) can be firm-specific to yield closed form solutions. But in order to focus on the main point of the paper, the model features only two types of firms: old-economy firms and new-economy firms, which are denoted by subscripts o and n. When a relation holds for either old-economy firms or new-economy firms, the subscript

[5]The production function here does not measure the quantity the firm can produce. It measures how much the firm can sell, so the factors from the demand side and from competition with peers are also coded in A_t. The productive technology is not openly available. A firm is established once the technology becomes known to its owner exogenously and the net present value of establishing the firm is positive.

[6]There can be other differences between old- and new-economy firms in addressing other issues.

is omitted. The distinction between old-economy firms and new-economy firms are characterized by (i) $\alpha_o = \alpha_n = \alpha$, $\beta_o < \beta_n$, and $\theta_o > \theta_n$ and (ii) $\beta_n - \beta_o = \theta_o - \theta_n$. Condition (i) says new-economy firms has the same productivity of tangible capital, higher productivity of intangible capital, but lower productivity of labor and raw materials. Condition (ii) leaves $\delta_o = \delta_n$, which makes the model more tractable.

In the model, all the firms are old-economy firms to begin with. Each year, a small fraction λ of old firms learns the productive technology of the new-economy firms and decide to switch to be new-economy firms, if the valuation of a new-economy firm is higher.

The dividend at period $t+1$ of a firm is $A_{t+1}K_t^{\alpha}N_t^{\beta}L_{t+1}^{\theta} - wL_{t+1} - I_{t+1} - J_{t+1}$. The firm maximizes its firm value as the sum of the expected, discounted future dividends, minus its set-up cost:

$$\max_{K_0, N_0, I_t, J_t, L_t, t\geq 1} E_0 \sum_{t=0}^{\infty} \phi^{t+1}(A_{t+1}K_t^{\alpha}N_t^{\beta}L_{t+1}^{\theta} - wL_{t+1} - I_{t+1} - J_{t+1}) - K_0 - N_0, \tag{7}$$

where $\phi \in (0,1)$ is the one-period discount factor for the firm, determined by the riskiness of dividends represented by the randomness of A_{t+1} and all other parameters mentioned above. In general, since the expected and the riskiness of the future dividends of old-economy firms and new-economy firms are different, the discount rates for the two types of firms can be different as well. In order to focus on the main issue of the paper, however, it is assumed here that the discount rates are the same.

In the model, $A_{t+1}K_t^{\alpha}N_t^{\beta}L_t^{\theta}$ is interpreted as sales, $A_{t+1}K_t^{\alpha}N_t^{\beta}L_{t+1}^{\theta} - wL_{t+1}$ is interpreted as gross profit, $A_{t+1}K_t^{\alpha}N_t^{\beta}L_{t+1}^{\theta} - wL_{t+1} - J_{t+1}$ is interpreted as operating income or cash flow (assuming the miscellaneous item is zero), which is further divided between tangible investment I_{t+1} and dividend in $t+1$. The fixed cost of production is not involved in the model. The model admits closed form solutions with derivations outlined in the Appendix. Relevant implications are stated below with brief explanations.

Implication 1. The optimal capital ratio of a firm, K_t/N_t, is strictly increasing in α and decreasing in β. It also strictly increases in ρ and decreases in ν_t, reflecting the effect of the durability of the respective type of capital. Old-economy firms have a higher K_t/N_t ratio than new-economy firms on average.

This is intuitive. The K_t/N_t ratio is strictly increasing with α and decreases with β, reflecting the effect of the marginal productivity of the respective type of capital. It is intuitive that the K_t/N_t decreases in the mean of ν_t, as a larger average ν_t means N_t is more durable, so the firm should keep a higher level of average N_t. In a particular year t, a higher ν_t means it's less costly to replenish intangible capital and therefore it's more worthwhile to go over the average.

Implication 2. Under the regularity condition: $\frac{\phi\beta_n}{1-\phi\bar{\nu}}e^{-\frac{\sigma_\varepsilon^2}{2(1-\theta_n)^2}} \geq 1$, $V_n > V_o$.

The regularity condition requires that the new-economy firms have high productivity in intangible capital (and low productivity in labor and raw materials), the intangible capital has high durability, the discount factor is high, and volatility in demand is not too high.

The optimal investment of tangible capital can be derived from $I_t = K_t - \rho K_{t-1}$. Similarly the optimal investment of intangible capital can be derived from $J_t = N_t - \nu_{t-1}N_{t-1}$.

Implication 3. Conditioned on $t-1$ information, investment in tangible capital, I_t, and intangible capital, J_t, are both linear function of sales, S_t (with the intercept and the slope coefficient being functions of $t-1$ variables).

$$I_t = g_{t-1}S_t - \rho K_{t-1}, \quad (8)$$

$$J_t = h_{t-1}S_t - \nu_{t-1}N_{t-1}. \quad (9)$$

An econometrician in the model is a person who observes all the parameters and variables except for ν_t. The econometrician has two tasks. The first is to explain I_t with all he observes up to t. The second is to predict C_{t+1} with all he observes at t.

The econometrician can explain I_t using S_t with $\hat{I}_t = \hat{g}_{t-1}S_t - \rho K_{t-1}$ where $\hat{g}_{t-1}$ is the estimate of g_{t-1} conditioned on the information at $t-1$. However, if the econometrician explains investment of tangible capital I_t using cash flow, an error occurs, because cash flow can be written as

$$C_t = (1-\theta-h_{t-1})S_t + \nu_{t-1}N_{t-1}. \quad (10)$$

where ν_{t-1} is unobserved by the econometrician. When S_t is replaced by cash flow in the econometrist's model, a noise term related to $\nu_t N_t$ is introduced.

Implication 4. Conditioned on C_t (and the disclosed variables available at $t-1$), an econometrician's best linear forecast of I_t contains a positive forecast error variance.

Implication 5. (i) S_{t+1} is best predicted by S_t, naturally. (ii) C_{t+1} is also better predicted by S_t, rather than by C_t.

The model presented here captures certain features which can be examined in the empirical work below. Strong assumptions are used in order to derive closed form solutions. The model is not meant to describe all the important features of the observed variables related to investment, cash flow and sales, but rather to pinpoint the major difference between sales and cash flow in explaining the investment in tangible capital. One key difference between the model here and the empirical work in the literature is that the equations in the model are conditional on information set, while the empirical work is typically unconditional. Another important difference is that the model here involves potentially different parameters for different type of firms, while in typical empirical work, the slope coefficient of the main variable of interest is a constant for all firms and only the intercept term is allowed to vary with firm through the use of firm fixed effect.

4. Main Empirical Results

I first examine investment regressions of the following type

$$\mathrm{Inv}_{it} = a_1 \mathrm{CF}_{it} + a_2 \mathrm{SA}_{it} + \gamma_i + \delta_t + \varepsilon_{it}. \tag{11}$$

where a_1 measures the investment-cash flow sensitivity, a_2 measures the investment-sales sensitivity, and ε_{it} is the generic term for regression errors. To focus on the time-variation of these sensitivities, the regressions are run for each of the five decades of the sample period separately. As a standard practice in the literature of the investment-cash flow sensitivity, both firm and year fixed effects (γ_i and δ_t, respectively) are included in the regressions and t-ratios of the parameters are based on standard errors that cluster by both firm and year. Table 2 reports the results of the investment regressions. Only slope coefficients are reported, as the fixed effects are of less interest.

Table 1 here

The first part of Table 1 reports the investment-cash flow sensitivity. The significant investment-cash flow sensitivity in the early decades and its decline in later decades have been widely documented, in Allayannis and Mozumdar (2004), Chen and Chen (2012), and Wang and Zhang (2021), for example, in their respective samples. In the literature, the investment regressions are usually run with the market-to-book asset ratio (i.e., the average Q) as one explanatory variable. While its slope coefficient is marginally, statistically significant, it is economically insignificant, as shown in many studies such as those cited above. Moreover, it does not affect the explanatory power of other potential variables that have been considered. For that reason, it is not included here to avoid distraction.

The middle part of Table 1 contains the main result of the investment-sales sensitivity. As seen in the table, it has been statistically and economically strong throughout the five decades of the sample period.[7] The last part of Table 1 reports the investment-cash flow sensitivity and the investment-sales sensitivity in multiple regressions. In the early decades, cash flow and sales share the predictive power. By the 1990s, sales have taken over cash flow in explaining investment. The coefficients of cash flow in the last three decades are negative, indicating that, compared with other components of sales, cash flow is less important in guiding investment decisions. The qualitative nature of the results are robust to inclusion of many other potential explanatory variables.

Table 2 reports the results of the regressions of the form

$$\mathrm{Inv}_{it} = a_1 \mathrm{OI}_{it} + a_2 \mathrm{GP}_{it} + \gamma_i + \delta_t + \varepsilon_{it}. \qquad (12)$$

[7] The slope coefficient in the last decade appears to be smaller than those in earlier decades. Part of the reason is that the standard deviation of investment becomes smaller in the last decade.

where OI is operating income and GP is gross profits. The purpose of showing the results is to demonstrate that the same patterns as revealed in Table 2 also manifest in OI and GP. OI is numerically close to CF. It is no wonder that the sensitivity of investment to OI is like that to CF: strong in early years and weakened in later years. GP is numerically quite different from SA, but it behaves like SA. What's common to CF and OI is that both are net income of the fixed cost, XSGA, and what's common to SA and GP is that both are income before XSGA.

Table 2 here

The main point of the paper is that current sales contain the information about current productivity, which predicts future productivity, future sales and future cash flows, while current cash flow, affected by current costs, especially the fixed cost for new-economy firms in later years, contains less information about future sales and future cash flows. This can be shown more directly by the results of the following regressions.

$$\mathrm{CF}_{it} = c_1\mathrm{CF}_{i,t-1} + c_2\mathrm{SA}_{i,t-1} + \gamma_i + \delta_t + \varepsilon_{it}, \tag{13}$$

$$\mathrm{SA}_{it} = d_1\mathrm{CF}_{i,t-1} + d_2\mathrm{SA}_{i,t-1} + \gamma_i + \delta_t + \varepsilon_{it}. \tag{14}$$

The models are definitely not the best models for predicting cash flow and sales. For example, many lagged firm characteristics can predict sales, as shown in Table 6. In particular, the relationships can last for more than one year. However, (13)-(14) are the simplest in conveying the point. Table 3 reports the results for the cash flow regression (13). It is conducted for the five ten-year subsamples, so we can track the changes over time. When lagged cash flow alone is used to predict cash flow, the slope coefficient is statistically significant, but the magnitude and significance are declining non-monotonically over time. When lagged sales alone are used to predict cash flow, the slope coefficient is also statistically significant, and the magnitude is increasing over time and statistical significance remains strong. When both lagged cash flow and sales are used to predict cash flow, sales do not help much in the first decade. But from the second decade on, while cash flow can still be predicted by lagged cash flow, lagged sales become a better predictor. Therefore, an explanation of the results in Table 1 is straightforward. Since cash flow loses its predictive power for future cash flows to sales in later decades, it is just natural that investment becomes sensitive to sales instead of cash flow.

Table 3 here

Table 4 reports the sales regressions. When lagged cash flow alone is used to predict sales, the slope coefficient is statistically significant in the first three decades, but the magnitude and significance decline monotonically over time. When lagged sales alone are used to predict sales, the predictive power is the strongest in the first decade. Afterwards, the predictive power is reduced, but remain stable and strong. When both lagged cash flow and sales are used to predict sales, it is lagged sales that positively predict sales. Except for the first decade of the sample, information about productivity is mainly contained in sales.

Table 4 here

The results presented in Tables 3 and 4 provide the reason why investment initially depends on cash flow, but more and more on sales. The negative coefficient of the lagged cash flow in the investment regressions when both lagged cash flow and sales are present deserves explanation. There is an economic reason of why the coefficient is negative, beside potential collinearity between cash flow and sales. For firms with the same lagged sales, it is the firms with lower cash flows that have higher current sales. This is so because the firms with lower lagged cash flows are firms with higher lagged expenses on intangible capital, which increase productivity.

5. Conclusions

I develop a model to explain why corporate investment is sensitive to sales and why it used to be sensitive to cash flow, but not any more. The model builds on the empirical observation that sales are persistent over time, but the information about future sales contained in current sales trickles down from sales to gross profit, to net income and then to cash flow. The largest drop in persistence occurs when the gross profit minus the sales, general and administrative expenses (XSGA) to reach net income. The time-series and cross-sectional volatility of XSGA is the culprit of the decline in the investment-cash flow sensitivity. This is more evident for new-economy firms.

The model assumes that the durability (i.e. one minus depreciation rate) of existing intangible capital is random. This randomness causes the randomness of the new investment in intangible capital, because it is optimal for firms to increase their investment in intangible when the durability has a negative shock, and vice versa. This induces a transitory shock to cash flow, diminishing its relation with the investment in tangible capital.

The contribution of the paper is its potential to settle the debate surrounding the cause of the investment-cash flow sensitivity between the proponents of the financial constraint explanation and the Q-theory explanation. The debate nearly ended without a winner when the investment-cash flow sensitivity disappeared around 2000. The finding of the investment-sales sensitivity, its theoretical foundation, and the associated evidence shed light on the issue which has puzzled financial economists for more than forty years.

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Table 1
Panel regressions of investment on cash flow and sales

This table reports the result of panel regressions of investment on cash flow and sales with firm and year fixed effects for all non-service firms in five 10-year subperiods of the sample 1975-2024. The dependent variable is tangible investment, Inv_t. The independent variable is cash flow (CF) and sales (SA). The numbers in parentheses are t-ratios clustering at firm and year. *, **, and *** indicates the p-values of the estimated coefficient being smaller than 0.1, 0.05, and 0.01, respectively. N is the number of firm-year observations. R^2 is the proportion of explained sample variance of the raw dependent variable by independent variables including fixed effects. R^2_* is the proportion of explained sample variance of the dependent variable net of fixed effects by independent variables excluding fixed effects.

	1975-1984	1985-1994	1995-2004	2005-2014	2015-2024
A. Inv_{it} on CF_{it} only					
CF	0.288***	0.065***	0.008	0.025***	0.000
	(10.99)	(6.08)	(0.76)	(4.15)	(0.26)
N	34458	41776	49750	36513	35513
R^2	0.593	0.521	0.582	0.672	0.657
R^2_*	0.063	0.011	0.000	0.004	0.000
B. Inv_{it} on SA_{it} only					
SA	0.045***	0.043***	0.039***	0.031***	0.030***
	(9.93)	(17.02)	(19.32)	(12.39)	(13.71)
N	34458	41776	49750	36513	35513
R^2	0.591	0.548	0.613	0.684	0.677
R^2_*	0.059	0.067	0.074	0.042	0.060
C. Inv_{it} on both CF_{it} and SA_{it}					
CF	0.202***	0.006	-0.023**	0.001	-0.005***
	(8.59)	(0.75)	(-3.09)	(0.16)	(-3.47)
SA	0.030***	0.042***	0.041***	0.031***	0.030***
	(7.05)	(18.44)	(17.48)	(10.59)	(13.68)
N	34458	41776	49750	36513	35513
R^2	0.602	0.548	0.614	0.684	0.677
R^2_*	0.084	0.067	0.077	0.042	0.061

Table 2
Panel regressions of investment on operating income and gross profit

This table reports the result of panel regressions of investment on operating income and gross profit. with firm and year fixed effects for all non-service firms in five 10-year subperiods of the sample 1975-2024. The dependent variable is tangible investment, Inv_t. The independent variable is operating income (OI) and gross profit (GP). The numbers in parentheses are t-ratios clustering at firm and year. *, **, and *** indicates the p-value of the estimated coefficient being smaller than 0.1, 0.05, and 0.01, respectively. N is the number of firm-year observations. R^2 is the proportion of explained sample variance of the raw dependent variable by independent variables including fixed effects. R^2_* is the proportion of explained sample variance of the dependent variable net of fixed effects by independent variables excluding fixed effects.

	1975-1984	1985-1994	1995-2004	2005-2014	2015-2024
A. Inv_{it} on OI_{it} only					
OI	0.205***	0.085***	0.028	0.038***	0.005
	(15.53)	(7.09)	(1.73)	(5.27)	(1.68)
N	34456	41775	49750	36513	35513
R^2	0.588	0.524	0.583	0.673	0.657
R^2_*	0.052	0.018	0.004	0.007	0.001
B. Inv_{it} on GP_{it} only					
GP	0.154***	0.109***	0.082***	0.068***	0.041***
	(13.76)	(14.55)	(14.17)	(15.16)	(9.30)
N	34456	41775	49750	36513	35513
R^2	0.598	0.546	0.606	0.683	0.668
R^2_*	0.076	0.063	0.057	0.039	0.034
C. Inv_{it} on both OI_{it} and GP_{it}					
OI	0.028	-0.046***	-0.061***	-0.026**	-0.012***
	(1.29)	(-4.42)	(-3.81)	(-2.63)	(-5.89)
GP	0.140***	0.130***	0.108***	0.080***	0.046***
	(7.64)	(14.81)	(11.97)	(10.88)	(9.44)
N	34456	41775	49750	36513	35513
R^2	0.598	0.547	0.610	0.684	0.670
R^2_*	0.076	0.066	0.068	0.041	0.038

Table 3
Panel regressions of cash flow on lagged cash flow and lagged sales

This table reports the result of panel regressions of current cash flow on lagged cash flow and lagged sales with firm and year fixed effects for all manufacturing firms in five 10-year subperiods of the sample 1975-2024. The dependent variable is current cash flow, CF_t. The independent variable is one-year lagged cash flow and sales. The numbers in parentheses are t-ratios clustering at firm and year. *, **, and *** indicates the p-value of the estimated coefficient being smaller than 0.1, 0.05, and 0.01, respectively. N is the number of firm-year observations. R^2 is the proportion of explained sample variance of the raw dependent variable by independent variables including fixed effects. R^2_* is the proportion of explained sample variance of the dependent variable net of fixed effects by independent variables excluding fixed effects.

	1975-1984	1985-1994	1995-2004	2005-2014	2015-2024
A. $\mathbf{CF}_{it}$ on $\mathbf{CF}_{i,t-1}$ only					
CF	0.294***	0.177***	0.175***	0.201***	0.147**
	(8.92)	(5.64)	(4.50)	(5.20)	(2.63)
N	26839	32225	38764	30794	28632
R^2	0.666	0.635	0.671	0.715	0.783
R^2_*	0.078	0.030	0.031	0.044	0.021
B. $\mathbf{CF}_{it}$ on $\mathbf{SA}_{i,t-1}$ only					
SA	0.027***	0.038***	0.047***	0.058***	0.069***
	(9.52)	(9.33)	(9.20)	(12.33)	(9.60)
N	26839	32225	38764	30794	28632
R^2	0.649	0.633	0.670	0.712	0.781
R^2_*	0.034	0.026	0.026	0.036	0.016
C. $\mathbf{CF}_{it}$ on both $\mathbf{CF}_{i,t-1}$ and $\mathbf{SA}_{i,t-1}$					
CF	0.263***	0.131***	0.138***	0.157***	0.130**
	(6.97)	(4.37)	(3.81)	(3.87)	(2.33)
Sale	0.009**	0.025***	0.034***	0.040***	0.058***
	(2.69)	(11.93)	(13.11)	(8.60)	(8.45)
N	26839	32225	38764	30794	28632
R^2	0.667	0.638	0.676	0.719	0.785
R^2_*	0.081	0.040	0.043	0.059	0.032

Table 4
Panel regressions of sales on lagged cash flow and lagged sales

This table reports the result of panel regressions of current sales on lagged cash flow and lagged sales with firm and year fixed effects for all manufacturing firms in five 10-year subperiods of the sample 1975-2024. The dependent variable is current sales, SA_t. The independent variable is one-year lagged cash flow and sales. The numbers in parentheses are t-ratios clustering at firm and year. *, **, and *** indicates the p-values of the estimated coefficient being smaller than 0.1, 0.05, and 0.01, respectively. N is the number of firm-year observations. R^2 is the proportion of explained sample variance of the raw dependent variable by independent variables including fixed effects. R^2_* is the proportion of explained sample variance of the dependent variable net of fixed effects by independent variables excluding fixed effects.

	1975-1984	1985-1994	1995-2004	2005-2014	2015-2024
A. $\mathbf{SA}_{it}$ on $\mathbf{CF}_{i,t-1}$ only					
CF	1.014***	0.350***	0.264***	0.078	0.017
	(8.03)	(5.82)	(4.75)	(1.38)	(0.48)
N	26839	32225	38764	30794	28632
R^2	0.876	0.816	0.797	0.862	0.864
R^2_*	0.021	0.007	0.007	0.001	0.000
B. $\mathbf{SA}_{it}$ on $\mathbf{SA}_{i,t-1}$ only					
SA	0.350***	0.244***	0.253***	0.298***	0.295***
	(11.10)	(7.26)	(5.86)	(6.66)	(5.82)
N	26839	32225	38764	30794	28632
R^2	0.890	0.827	0.810	0.876	0.877
R^2_*	0.127	0.066	0.070	0.104	0.096
C. $\mathbf{SA}_{it}$ on both $\mathbf{CF}_{i,t-1}$ and $\mathbf{SA}_{i,t-1}$					
CF	-0.294**	-0.119*	-0.006	-0.282***	-0.072**
	(-2.91)	(-1.91)	(-0.13)	(-6.60)	(-2.50)
SA	0.371***	0.256***	0.253***	0.330***	0.301***
	(10.44)	(6.85)	(5.63)	(7.30)	(6.01)
N	26839	32225	38764	30794	28632
R^2	0.890	0.827	0.810	0.877	0.878
R^2_*	0.128	0.067	0.070	0.113	0.097

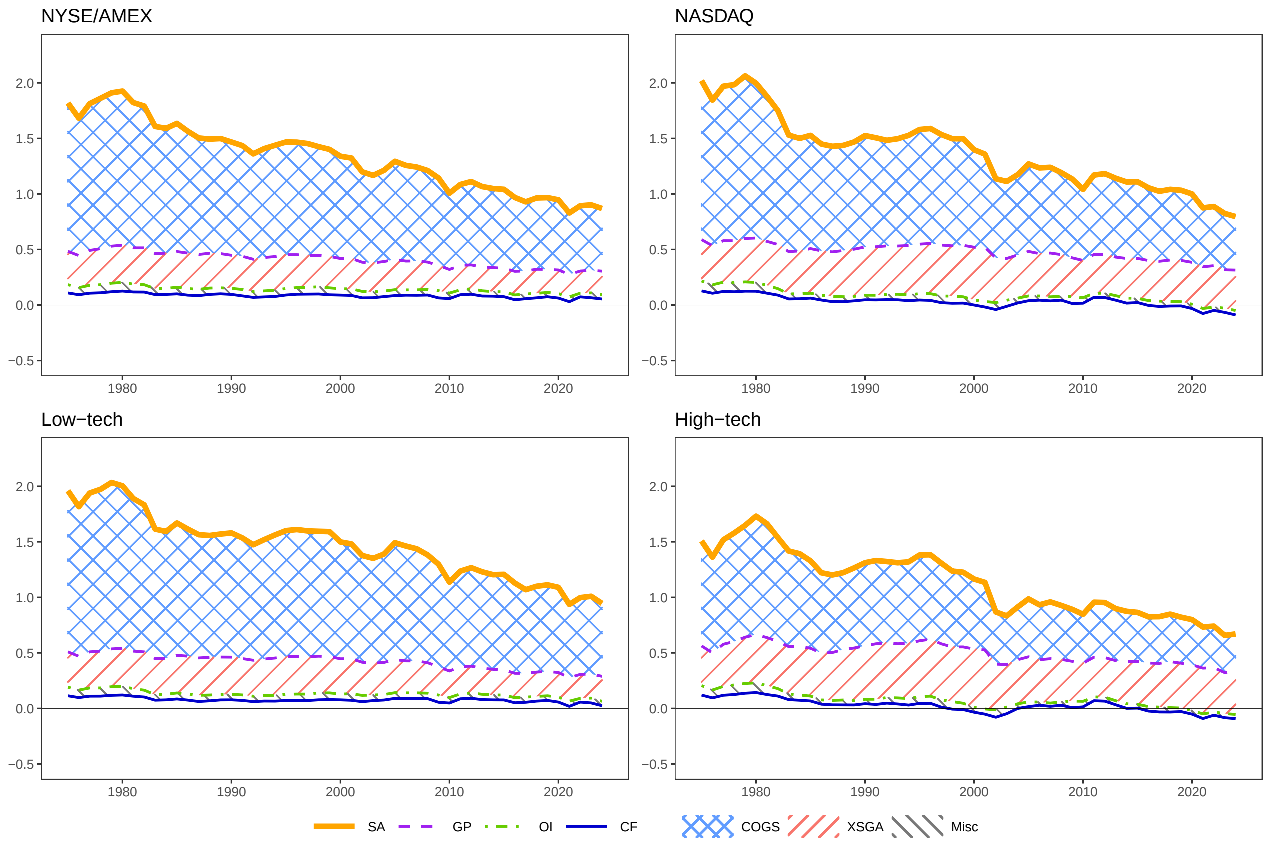

Figure 1. Cross-sectional averages of SA, GP, OI, CF, COGS, XSGA, and Misc
The figure plots the annual time series of the cross-sectional averages of sales (SA), gross profit (GP), operating income (OI) and cash flow (CF) in lines, and cost-of-goods-sold (COGS), selling, general and administrational expenses (XSGA) and miscellaneous items (Misc), in shaded areas. The time period is 1975-2024. Each variable is scaled by firms' total assets. The cross-sectional average each year is taken with the distribution winsorized at 1% and 99%.

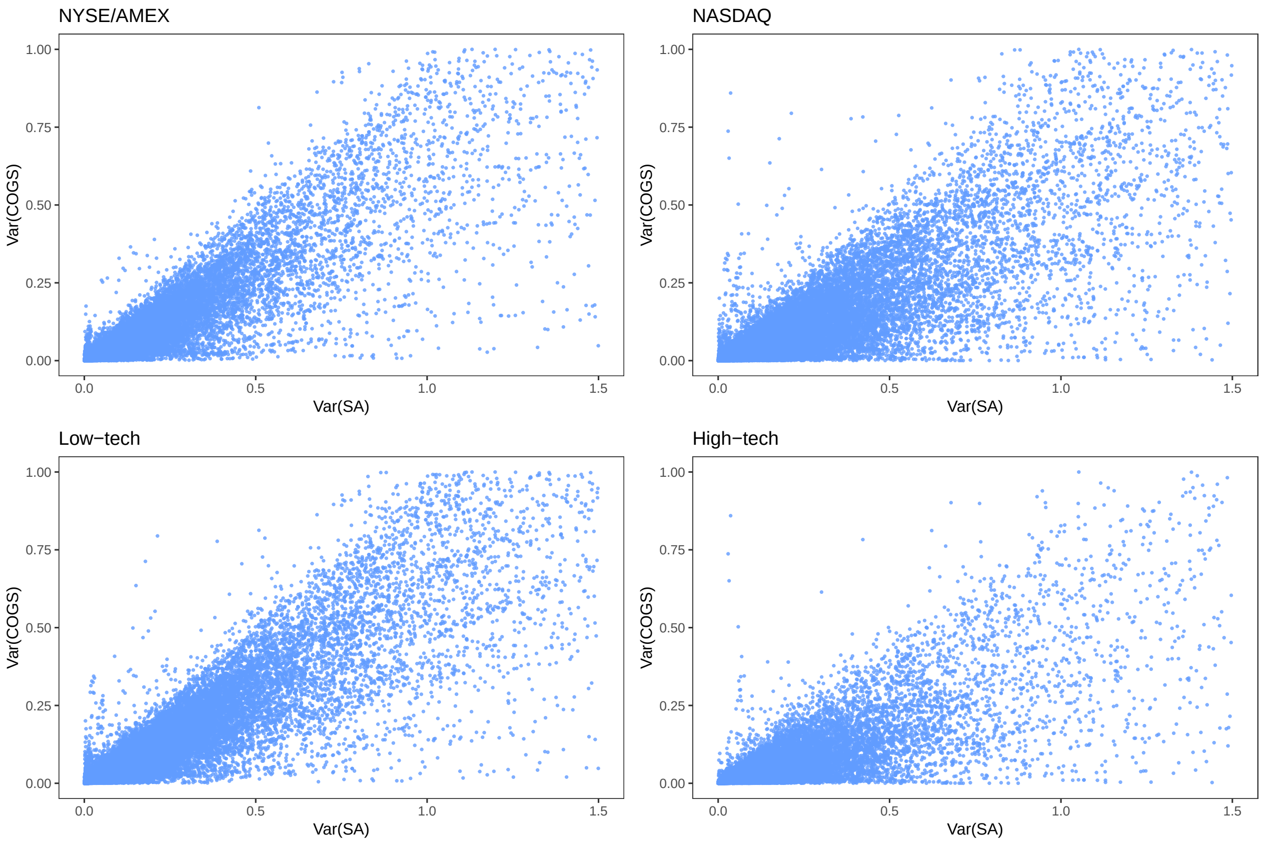

Figure 2. Scatter plot of Var_{it}(COGS) vs. Var_{it}(SA)

The figure is a scatter plot of Var_{it}(COGS) vs. Var_{it}(SA). Each point is a firm/year observation, whose vertical coordinate is the sample variance of COGS and horizontal coordinate is the sample variance of SA, both calculated for the firm/year with the past ten years of the variable.

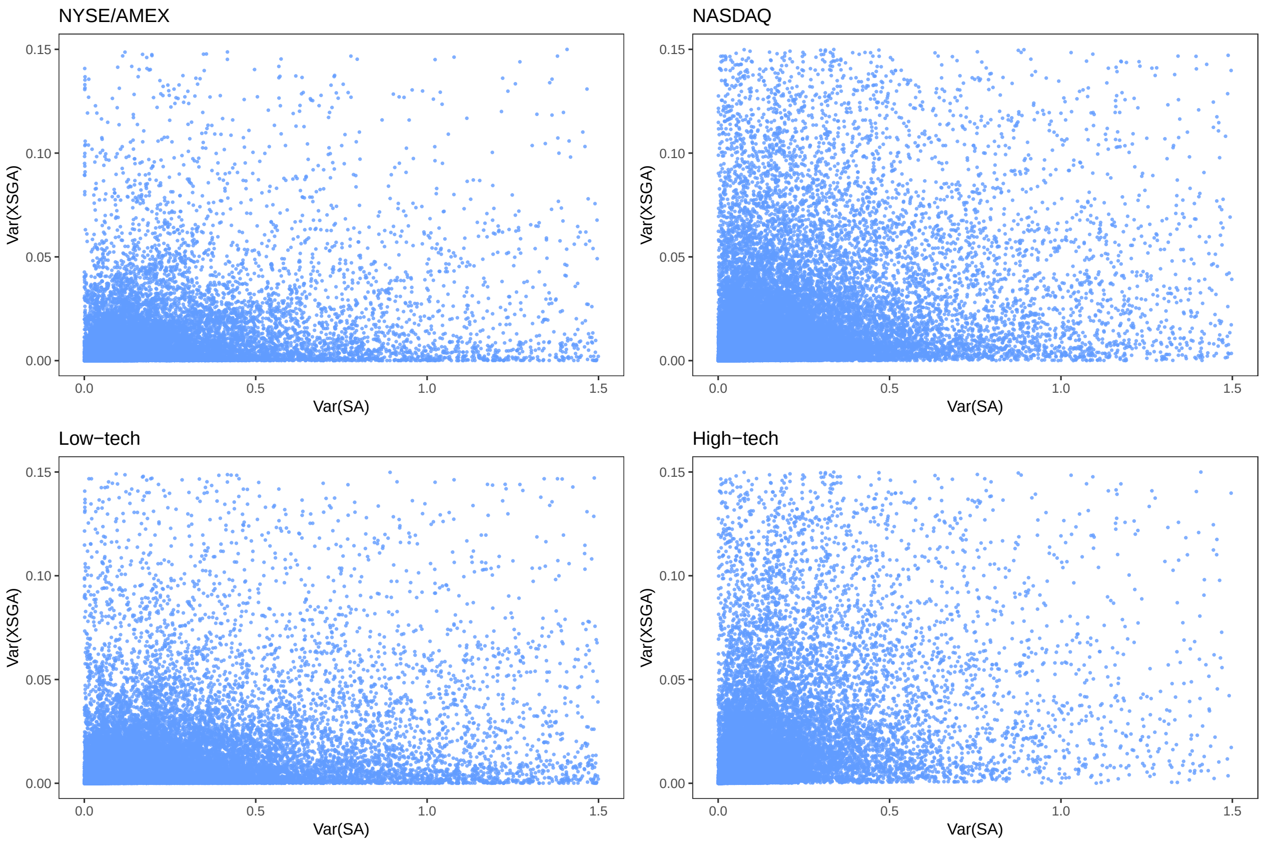

Figure 3. Scatter plot of Var_{it}(XSGA) vs. Var_{it}(SA)

The figure is a scatter plot of Var_{it}(COGS) vs. Var_{it}(SA). Each point is a firm/year observation, whose vertical coordinate is the sample variance of COGS and horizontal coordinate is the sample variance of SA, both calculated for the firm/year with the past ten years of the variable.

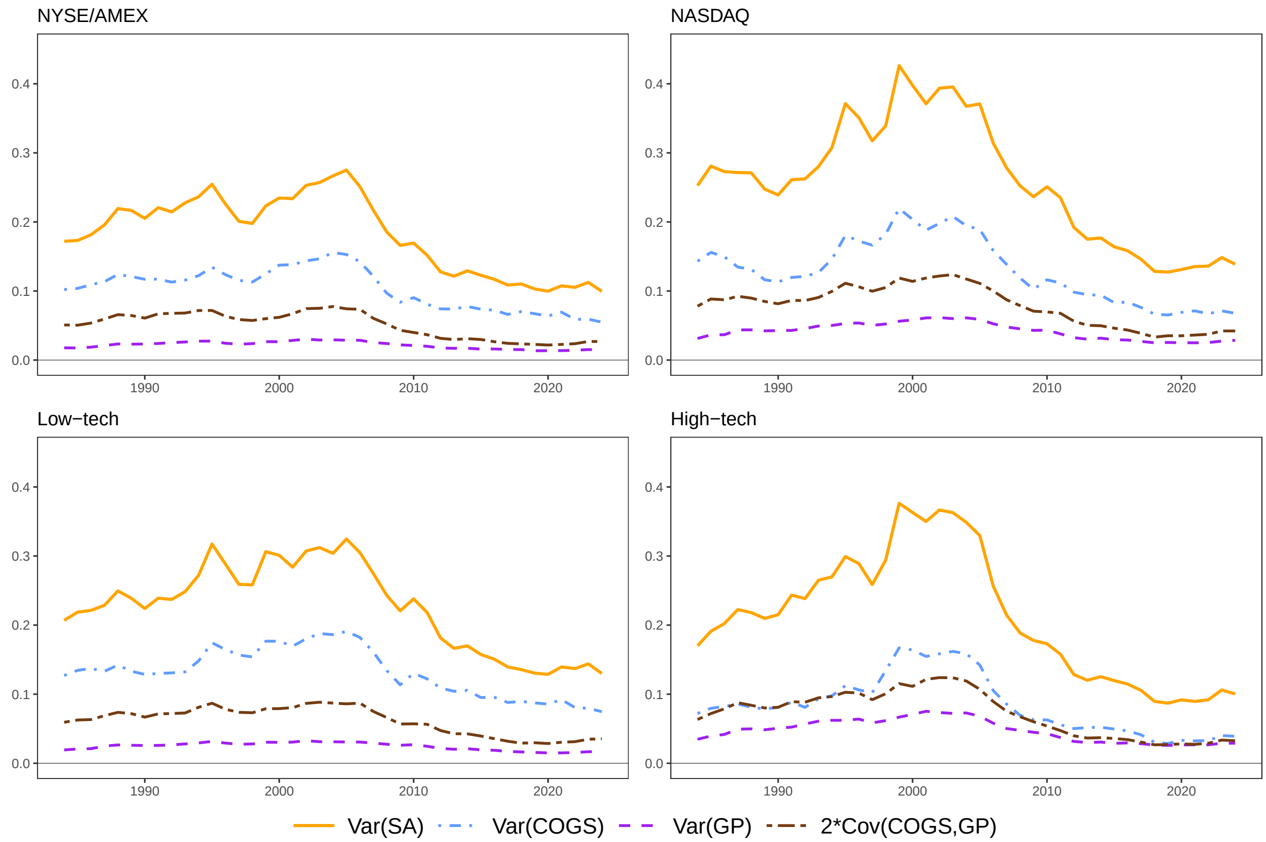

Figure 4. Cross-sectional averages of sample variances of SA, COGS, and GP
The figure plots the annual time series of the cross-sectional averages of the sample variance of asset-scaled sales (SA), cost-of-goods-sold (COGS), and gross profit (GP). It also plots the annual time series of the cross-sectional averages of the sample covariance between COGS and GP. The time period is 1984-2024. The sample variances and covariance are calculated with data in the past ten years. The cross-sectional average each year is taken with the distribution winsorized at 1% and 99%.

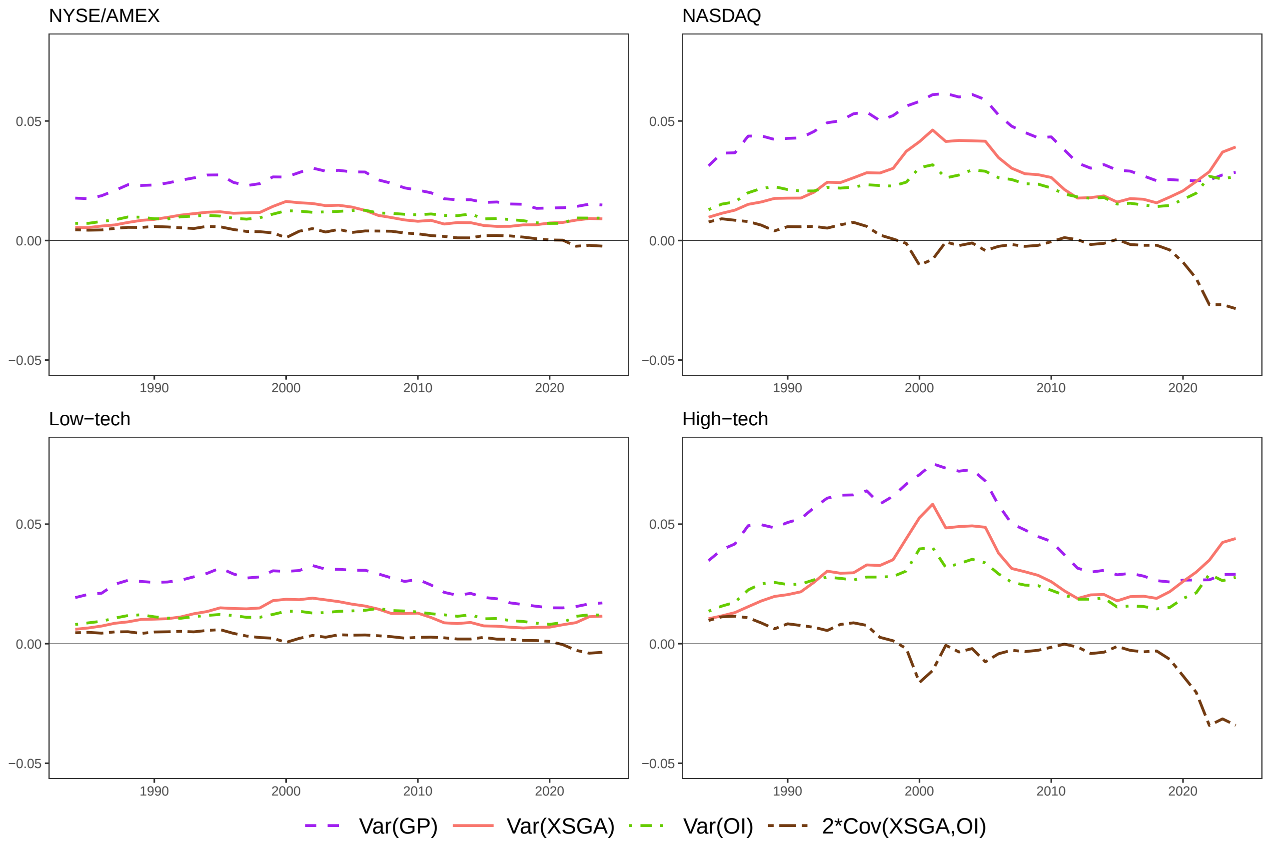

Figure 5. Cross-sectional averages of sample variances of GP, XSGA, and OI

The figure plots the annual time series of the cross-sectional averages of the sample variance of asset-scaled gross profit (GP), selling, general and administrational expenses (XSGA), and operating income (OI). It also plots the sample covariance between XSGA and OI. The time period is 1984-2024. The sample variances and covariance are calculated for each firm-year with data in the past ten years. The cross-sectional average each year is taken with the distribution winsorized at 1% and 99%.

Appendix A: Model Derivation

The derivation of the model for the optimal investment in tangible capital and intangible capital is the same for old-economy and new-economy firms. Therefore, much of it can be combined without specific mentioning of the firm type.

The optimal L_t is determined by the first-order condition of the one-period production function: $\frac{\partial}{\partial L_t} A_t K_t^a N_t^b L_t^c = w$, as a function of K_t and N_t. When the optimal L_t, which satisfies $wL_t = cS_t$, is substituted in (7), the problem becomes

$$
\begin{aligned}
& \max_{I_t, J_t, t \geq 0} E_0 \sum_{t=0}^{\infty} \phi^t \left(A_t^* K_t^\alpha N_t^\beta - I_t - J_t \right) && (15) \\
= & \max_{K_t, N_t, t > 0} E_0 \sum_{t=0}^{\infty} \phi^t \left(A_t^* K_t^\alpha N_t^\beta - K_{t+1} + \kappa K_t - N_{t+1} + \nu_t N_t \right), && (16)
\end{aligned}
$$

where $A_t^* = (1-c)\left(\frac{c}{w}\right)^{\frac{c}{1-c}} A_t^{\frac{1}{1-c}} \equiv c^* A_t^{\frac{1}{1-c}}$, $\alpha = a/(1-c)$ and $\beta = b/(1-c)$ with $\alpha + \beta < 1$, and I_t and J_t are substituted out using (5) and (6). $A_t^* K_t^\alpha N_t^\beta = S_t - wL_t = (1-c)S_t$ here is interpreted as gross profit, i.e., sales minus the variable cost. Once $\{K_t\}_{t=1}^{\infty}$ and $\{N_t\}_{t=1}^{\infty}$ are decided, optimal investment I_t and J_t can be derived from (5) and (6).

The optimal K_t and N_t can be solved jointly as

$$
\begin{aligned}
K_t &= \left\{ \phi \alpha^{*(1-\beta^*)} \beta^{*\beta^*} \left(\frac{1}{1-\phi\rho} \right)^{1-\beta^*} \left(\frac{1}{1-\phi\nu_t} \right)^{\beta^*} E_t A_{t+1}^* \right\}^{\frac{1}{1-\alpha^*-\beta^*}}, && (17) \\
N_t &= \left\{ \phi \alpha^{*\alpha^*} \beta^{*(1-\alpha^*)} \left(\frac{1}{1-\phi\rho} \right)^{\alpha^*} \left(\frac{1}{1-\phi\nu_t} \right)^{1-\alpha^*} E_t A_{t+1}^* \right\}^{\frac{1}{1-\alpha^*-\beta^*}}. && (18)
\end{aligned}
$$

Substituting this to (16) gives the value of the firm as

$$
\begin{aligned}
V &= (1-\alpha^*-\beta^*) \left[\left(\frac{\alpha^*}{1-\phi\rho} \right)^{\alpha^*} \left(\frac{\beta^*}{1-\phi\bar{\nu}} \right)^{\beta^*} \phi\theta^* \exp\left(\frac{2\eta(1-\theta)+\sigma_\varepsilon^2}{2(1-\theta)^2} \right) \right]^{\frac{1}{1-\alpha^*-\beta^*}} \\
& \quad \cdot E_0 \sum_{t=0}^{\infty} \phi^t A_t, && (19)
\end{aligned}
$$

Proof of Implication 1. From (17)-(18), the optimal capital ratio is

$$
\frac{K_t}{N_t} = \frac{\alpha^*}{\beta^*} \frac{1-\phi\nu_t}{1-\phi\rho} = \frac{\alpha}{\beta} \frac{1-\phi\nu_t}{1-\phi\rho}
$$

The statement for old- and new-economy firms follows from Condition (i) that $\alpha_o = \alpha_n$ and $\beta_o < \beta_n$.

Proof of Implication 2. The new economy firm can be parameterized having $\alpha_n = \alpha_o = \alpha$, $\beta_n = \beta_o + \xi$ and $\theta_n = \theta_o - \xi$. Substituting these into the V function above generates the desired result with the regularity condition.

Proof of Implication 3. Both K_t and N_t contain a factor $(E_t A^*_{t+1})^{1/(1-\alpha^*-\beta^*}$, which reduces to a multiplier of A_t due to the assumption that S_{t+1} is linear homogeneous in (A_t, K_t, N_t, L_{t+1}). The implication follows from the fact $A_t = S_t/K_{t-1}N_{t-1}$.

Proof of Implication 4. This can be shown using the same logic as in the case when the error-in-variable (EIV) problem is present in the regression model. (10) is similar to EIV.

Proof of Implication 5. The same as Implication 4.

Appendix B:

Table A1

Definition of high-tech industries with exaples of constituent firms

This table lists the 3-digit SIC codes of high-tech industries, along with their shortened industry names and a few examples of firms belonging to the industries.

SIC	Industry name	Examples of firms
283	Drugs	Pfizer, J&J, Merck
355	Special Industry Machinery except Metalworking	ASML, Applied Materials, Lam Research
357	Computer and Office Equipment	IBM, Apple, Cisco, Dell, HP
366	Communications Equipment	Apple, Nokia, Motorola, Qualcomm
367	Electronic Components and Accessories	Nvidia, Broadcom, TSMC, AMD, Intel
372	Aircraft and Parts	Boeing, GE Aerospace
382	Laboratory Apparatus and Analytical, Optical Measuring, and Controlling Instruments	Emerson, KLA
384	Surgical, Medical, and Dental Instruments and Supplies	Abbott Lab, Medtronic
481	Telephone Communications	AT&T, Verizon, T-Mobile, Vodafone
483	Radio and Television Broadcasting Stations	Paramount, FOX
484	Cable and other Pay Television Services	Comcast, Time Warner
489	Communication Services, Not Elsewhere Classified	Echostar. Telesat
596	Nonstore Retailers	Amazon, Alibaba, JD
737	Computer Programming, Data Processing, and other Computer Related Services	Alphabet, Microsoft, Palantir, Spotify, Disney, Oracle, SAP, Adobe